

Chapter 1

Permanent Electric Dipole Moments of Single-, Two-, and Three-Nucleon Systems

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A nonzero electric dipole moment (EDM) of the neutron, proton, deuteron or helion, in fact, of any finite system necessarily involves the breaking of a symmetry, either by the presence of external fields (i.e. electric fields leading to the case of *induced* EDMs) or explicitly by the breaking of the discrete parity and time-reflection symmetries in the case of *permanent* EDMs. We discuss two theorems describing these phenomena and report about the cosmological motivation for an existence of *CP* breaking beyond what is generated by the Kobayashi-Maskawa mechanism in the Standard Model and what this might imply for the permanent electric dipole moments of the nucleon and light nuclei by estimating a window of opportunity for physics beyond what is currently known. Recent – and in the case of the deuteron even unpublished – results for the relevant matrix elements of nuclear EDM operators are presented and the relevance for disentangling underlying New Physics sources are discussed.

1. The Problem with Permanent Electric Dipole Moments

Gerry Brown was always interested in *magnetic* dipole moments of baryons and nuclei, and especially in confronting the theoretical predictions of quark, chiral bag and Skyrme models with the experimental results. But to our knowledge (compare also with Ref. 1), he never worked on *electric*

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dipole moments. This choice definitely turned out to be wise in his case, since during his lifetime – and even until the time of writing – all measurements of the electric dipole moment of any (sub-)atomic particle have been compatible with zero – only more and more restrictive lower bounds have been established since the first experiment in the fifties of the last century by Smith, Purcell, and Ramsey² for the neutron EDM.

1.1. The subtle character of EDMs of subatomic particles

Why are permanent electric dipole moments (EDMs), which in classical electrodynamics just correspond to (the integrals over) the spatial three-vectors of displaced charges (or in general charge densities), much more subtle in the case of subatomic particles or, generically, in the realm of Quantum Mechanics? The reason is that their existence is tied to the breaking of the discrete symmetries of parity (P) conservation and time-reflection (T) invariance, such that they are intrinsically small.^a In fact, the order of magnitude of the nucleon EDM (d_N) can be estimated as follows:³

- (I) The starting scale is given by the CP and P conserving (magnetic) moment of the nucleon, which is of the order of the nuclear magneton

$$\mu_N = e/(2m_p) \sim 10^{-14} e \text{ cm}, \quad (1)$$

where $e > 0$ is the unit of electric charge and m_p the proton mass.

- (II) Furthermore, as we will discuss below, a nonzero permanent EDM requires P and CP violation. The cost of P violation can empirically be estimated in terms of Fermi's constant $G_F \approx 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$ times the square of the axial decay constant of the pion, $F_\pi \approx 92.2 \text{ MeV}$, the order-parameter of the spontaneous breaking of chiral symmetry of Quantum Chromodynamics (QCD) at low energies.⁴ The dimensionless product scales therefore as $G_F \cdot F_\pi^2 \sim 10^{-7}$.
- (III) The cost related to the *additional* CP violation follows from, e.g., the ratio of the amplitude moduli of K_L^0 to K_S^0 decays into two pions:⁴

$$|\eta_{+-}| = \frac{|\mathcal{A}(K_L^0 \rightarrow \pi^+ \pi^-)|}{|\mathcal{A}(K_S^0 \rightarrow \pi^+ \pi^-)|} = (2.232 \pm 0.011) \cdot 10^{-3}. \quad (2)$$

In summary, the modulus of the EDM of the nucleon cannot be larger than

$$|d_N| \sim 10^{-3} \times 10^{-7} \times \mu_N \sim 10^{-24} e \text{ cm}, \quad (3)$$

^aThe breaking of T implies CP violation (in terms of the charge conjugation C symmetry) if CPT is conserved.

which is ten orders of magnitude smaller than the corresponding magnetic dipole moments, without getting into conflict with known physics — on top of the EDM measurements themselves which are nowadays even more restrictive (see below).

In the Standard Model the sole source for CP violation, if the QCD θ term is assumed to be absent, is the Kobayashi-Maskawa⁵ (KM) mechanism, which, however, only generates a nonzero CP violating phase if the Cabbibo-Kobayashi-Maskawa (CKM) quark-mixing matrix involves at least three quark generations. This KM-generated CP violation is therefore flavor-violating, while the EDMs are, by nature, flavor-diagonal. This means that the SM (without the QCD θ term) is “punished” by the additional cost of a further factor $G_F F_\pi^2 \sim 10^{-7}$ to undo the flavor violation. In summary, the SM prediction for the nucleon EDM, based on the KM mechanism, is therefore much smaller than Eq. (3), namely

$$|d_N^{\text{SM}}| \sim 10^{-7} \times 10^{-24} \text{ e cm} \sim 10^{-31} \text{ e cm}. \quad (4)$$

This result agrees in magnitude with the three-loop estimates of Refs. 6,7 and also with the two-loop calculations of Refs. 8,9 (see also Ref. 10) which include both a strong *penguin* short-range diagram and a long-range pion loop.^b Even recent *loop-less* calculations¹¹ involving propagators of charm-flavored sea-quarks give a result of the same order.^c

From the above estimates one can infer that an EDM of the nucleon measured in the window

$$10^{-24} \text{ e cm} > |d_N| \gtrsim 10^{-30} \text{ e cm} \quad (5)$$

could be a clear signal for new physics beyond the KM mechanism of the Standard Model: either *strong CP violation* by a sufficiently large QCD θ term or genuinely new physics, as, e.g., supersymmetric (SUSY) models, multi-Higgs models, left-right-symmetric models etc.

1.2. Two theorems for the existence of EDMs

This brings us back to the original question: Why do nonzero electric dipole moments of finite quantum systems necessarily require the breaking of some

^bNote that one-loop contributions to EDMs resulting from the KM mechanism of the SM have to vanish, since the CP -violating KM matrix element at the first loop vertex is canceled by its Hermitian conjugate at the other side.

^cThe EDM of the electron is even further suppressed by a factor 10^{-7} in the SM, i.e. $|d_e^{\text{SM}}| \sim 10^{-38} \text{ e cm}$, which follows from a further weak-interaction insertion and one additional quark/gluon loop.¹² The SM prediction for the muon is slightly larger, namely $|d_\mu^{\text{SM}}| \sim 10^{-35} \text{ e cm}$, because of the lepton mass ratio $m_\mu/m_e \sim 200$.

symmetry? The above statement can be interpreted as a special case of the following theorem which, e.g., is well-known to apply for the case of the chiral symmetry breaking for lattice QCD (see e.g. Refs. 13,14):^d

Theorem 1. *In any finite quantum system in the absence of any explicitly broken symmetry there cannot exist a spontaneously broken ground state.*

The condition of *finiteness* applies to both the spatial extent of the system and to the height of the pertinent quantum “walls”. Therefore the tunneling probability from *any* broken-symmetry state to any *alternative* one is non-vanishing. This opens up the way for finite systems to form one totally symmetrized state from all the broken-symmetry alternatives which is then the real ground state of the system, while the same tunneling amplitudes induce non-vanishing gaps to suitable antisymmetric combinations of these states which are therefore excited states and non-degenerate to the ground state.

The question might arise why this does not apply to magnetic moments which have known and very well-tabulated⁴ nonzero values for the cases of the electron, muon, proton, neutron, other baryons, nuclei etc. In fact, the theorem applies but the solution is trivial. The non-zero value of the total angular momentum (i.e. the spin in the case of subatomic particles) suffices to induce the (rotational) symmetry breaking since it defines an axis (in the laboratory frame) for the projection of the magnetic moment,^e which shares the same axial-vector properties as the spin. The appearance of a nonzero magnetic moment for a particle without any spin or angular momentum is forbidden and would indeed come as a surprise.

So what is the difference to the case of an electric dipole moment which has the operational definition of the displacement vector of the charges? Why is the existence of a nonzero spin of a particle or finite quantum system not enough to induce the necessary symmetry breaking? The difference is that spins and, in general, angular momenta \vec{J} are *axial* vectors (as the magnetic fields \vec{B}) while the electric dipole moment \vec{d} is a *polar* vector:

$$\begin{aligned} \vec{J} &\xrightarrow{P} \vec{J}, & \vec{d} &\xrightarrow{P} -\vec{d}, \\ \vec{J} &\xrightarrow{T} -\vec{J}, & \vec{d} &\xrightarrow{T} \vec{d}. \end{aligned} \tag{6}$$

^dIn a finite lattice scenario, even when the lattice size becomes larger and larger, a non-zero value of a quark condensate can only be measured if the mass of the pertinent current quark differs from zero corresponding to an *explicit* breaking of chiral symmetry.

^eMore precisely, for any non-zero spin and any direction in space it is always possible to find an eigenstate of the spin operator with *non-vanishing* projection (quantum number).

Without the explicit breaking of the discrete T (time-reflection) and P (parity) symmetries, the presence of a non-zero spin or angular momentum would not be enough to define the direction for the projection of the EDM vector, since the sign of this projection would be reversed under the above mentioned discrete symmetries if they are conserved. However, in the rest-frame of any subatomic particle with non-vanishing mass there are simply not any other vectors than the spin and total angular momentum. So it should be clear that at least in these cases there is a need for extra symmetry breaking if these particles are to carry a non-vanishing permanent EDM. The word *permanent* is important here, since the realization of an *induced* EDM is of course possible – without the breaking of T and P – in the presence of a non-vanishing electric field \vec{E} which has the properties of a polar vector as the EDM. Note that, interpreted in this way, our theorem still holds also for the case of *induced* EDMs: the presence of an electric field, which by nature breaks the (rotational) symmetry of the system, is the stated precondition.

But we know that macroscopic and mesoscopic devices (capacitors, batteries, etc.) and even certain molecules (H_2O or NH_3) obviously can have sizable dipole moments which correspond to their spatial extent times the involved charges. Well, most of these systems break a symmetry classically. There is e.g. a spatial vector pointing from one plate of the capacitor to the other one or from one nucleus in a diatomic polar molecule to the other one (which differs in charge and/or mass). But if these systems are interpreted quantum mechanically, as axially symmetric rods or as (a)symmetric tops, one should keep in mind the difference between body-fixed directions and lab-frame-fixed ones. If a polar symmetry still applies, the projection on a stationary state of fixed angular momentum in the lab-fixed frame suffices to average out the body-fixed (classical or intrinsic) EDM to a vanishing expectation value.^{3,15,16} In the (a)symmetric top scenario, the tunneling amplitude from the state pointing in one direction of the lab-frame (spin) axis to the one projected onto the opposite direction would be small, but nonzero,³ such that even then the theorem applies. A nonzero EDM is measured in practice in the latter cases, since the applied electric fields might be small but non-vanishing, such that the measured EDM has the character of an induced EDM. Alternatively, the pertinent temperature of the system is nonzero, such that the resulting de-facto degeneracy between the parity-even ground state and excited parity-odd states together with the direction defined either by the non-vanishing electric field or solely by the total-spin direction would be sufficient to define the orientation for the

resulting explicit symmetry breaking. If however the system were cooled down to such small temperatures and sufficiently shielded against electric fields that the de-facto degeneracy would be lifted, a non-vanishing EDM could not be measured even in these cases. Only a truly infinite system would escape the consequences of the theorem.

Let us summarize what was stated above by the following theorem which describes the existence of permanent EDMs:^{1,17}

Theorem 2. *Any non-vanishing coefficient d in the relation of the expectation values $\langle j^P | \vec{d} | j^P \rangle = d \langle j^P | \vec{J} | j^P \rangle$ of the electric dipole moment operator $\vec{d} \equiv \int \vec{r} \rho(\vec{r}) d^3r$ (where $\rho(\vec{r})$ is the charge density) and the total angular momentum \vec{J} expressed in terms of a stationary state $|j^P\rangle$ of a particle with at least one nonzero generalized ‘charge’, nonzero mass, nonzero total angular momentum j and specified parity P , such that $\langle j^P | \vec{J} | j^P \rangle \neq 0$ in general, and no other energy degeneracy than its rotational one is a signal of P and T violation^f and, because of the CPT theorem, of flavor-diagonal CP violation.*

The above particle can be an ‘elementary’ particle as a quark, charged lepton, W^\pm boson, Dirac neutrino, etc., or a ‘composite’ particle as a neutron, proton, nucleus, atom, molecule or even a solid body, as long as it meets the requirements stated in the above theorem. Namely, it is important that

- (i) the particle or system should carry a non-vanishing angular momentum to define an axis (excluding therefore scalar and pseudoscalar particles),
- (ii) it should not be self-conjugate^g in order to prevent that the charge-conjugation property of the particle does not even allow a unique orientation in its body-fixed frame,
- (iii) and that there should be no degeneracy (except the one related to the *magnetic* quantum number). Otherwise the ground state of even parity could mix with a state of opposite parity and the directional information coming from the spin would suffice to define the quantization axis for the EDM without the need of explicit P and T breaking.

However, it is well known and especially applies to the case of molecular systems with closely spaced rotational levels or atoms with a sizable octupole moment that the near-degeneracy of the two opposite-parity levels

^fWithout the violation of P , $\langle j^P | \vec{d} | j^P \rangle$ would just vanish since $\mathcal{P} | j^P \rangle = (-1)^P | j^P \rangle$ and $\mathcal{P} \vec{d} \mathcal{P} = -\vec{d}$, where \mathcal{P} is the parity operator which has the property $\mathcal{P}^2 = 1$.

^gExamples for *self-conjugate* particles with spin are Majorana neutrinos or the ω , ρ^0 or ϕ^0 mesons, but not their SU(3) partners K^* which carry strangeness quantum numbers.

might produce sizable enhancement factors for P and T violating quantities, see e.g. Refs. 15,16. A similar mechanism is at work in the case of the *induced* EDMs of water or ammonia molecules, see e.g. Ref. 18: In a simplified picture there is a pair of nearly degenerate states of opposite parity $|\pm\rangle$ (where $|+\rangle$ is the ground state) with energy levels which rearrange according to

$$E_{2,1} = \frac{1}{2}(E_- + E_+) \pm \sqrt{\frac{1}{4}(E_- - E_+)^2 + (e\langle\vec{r}\rangle \cdot \vec{E})^2} \quad (7)$$

when exposed to an electric field \vec{E} . Here $\langle\vec{r}\rangle$ is the *transition* (not a diagonal!) matrix element of the charge displacement vector \vec{r} between the states $|+\rangle$ and $|-\rangle$. For a sufficiently large \vec{E} field, the second term in the square root dominates and there will be an approximately linear behavior of the levels, $E_{2,1} = \frac{1}{2}(E_- + E_+) \pm |e\langle\vec{r}\rangle \cdot \vec{E}|$, which mimics a linear Stark effect – but note the appearance of an absolute value. For a weak enough \vec{E} -field and a sufficiently low temperature we would find instead the following behavior (quadratic Stark effect):

$$E_{2,1} = E_{\mp} \pm \frac{(e\langle\vec{r}\rangle \cdot \vec{E})^2}{E_- - E_+} + \dots \quad (8)$$

Thus the molecule has always an *induced* EDM which can be enhanced by the small energy difference between the states of opposite parity.^h

2. Motivation for EDMs

Why should we be interested in measuring *permanent* EDMs? One reason is of course the window of opportunity which the tiny CP -violating Kobayashi-Maskawa mechanism of the SM opens for the search of *New Physics*, see relation (5). The other reason is the CP violation by itself. Independently of how much matter surplus might have originally been created in the Big Bang, after the inflation epoch the primordial baryon–antibaryon (density) asymmetry should have been leveled out to an extremely high precision. However, about 380000 years later, when the temperature of the Universe had sufficiently decreased such that hydrogen atoms became stable against the radiation pressure and therefore the corresponding photons could not couple any longer to an electron-proton plasma, the ratio of this asymmetry to the photon density n_γ had the following measured value:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \Big|_{\text{CMB}} = (6.05 \pm 0.07) \cdot 10^{-10} . \quad (9)$$

^hThe case of a two-level system with a magnetic-moment interaction in the presence of a magnetic field is totally different, since there is always a linear contribution $\langle\pm|\vec{\mu}|\pm\rangle \cdot \vec{B}$, no matter how weak the \vec{B} field might be, because these expectation values are diagonal.

This result was derived from the cosmic microwave background (CMB) measurements by the COBE, WMAP and Planck satellite missions,¹⁹ while the prediction of the SMs of particle physics and cosmology is more than seven orders of magnitude less, see e.g. Ref. 20.

In fact, CP violation is one of the three conditions for the dynamical generation of the baryon–antibaryon asymmetry during the evolution of the universe as formulated by Andrey Sakharov in 1967.²¹ These conditions can be paraphrased as follows:

- (i) There has to exist a mechanism for the generation of baryon charge B in order to depart from the initial value $B = 0$ (after inflation).
- (ii) Both C and CP have to be violated such that the production mechanisms and rates of B can be distinguished from the ones of \bar{B} (even then the pertinent helicities are summed).
- (iii) Either CPT has to be broken as well¹ or the dynamical generation had to take place during a stage of non-equilibrium (i.e. a sufficiently strong first order phase transition) to discriminate, in the average, the B production reaction from its back reaction and to escape from the fact that $\langle B \rangle = 0$ holds on the average if CPT symmetry holds, i.e. from the time-independence in the equilibrium phase.

While baryon plus lepton number ($B + L$) violation can be accommodated by the Standard Model in an early stage of the evolution via sphalerons,²² the SM cannot satisfy the other two conditions:

- (i) the CP breaking by the Kobayashi–Maskawa (KM) mechanism⁵ is far too small; even a $\bar{\theta}$ angle¹ of the order of 10^{-10} which would still be compatible with the empirical bound for the neutron EDM²⁵ cannot help in generating a sufficient baryon–antibaryon asymmetry: there would be a mismatch of the scales relevant for an electroweak (or even higher) transition on the one hand and the $\sim 1\text{--}2$ GeV regime where QCD becomes sufficiently non-perturbative such that instanton effects are not suppressed any longer on the other hand, see e.g. Ref. 26;
- (ii) at vanishing chemical potential the SM, which as a relativistic quantum field theory is of course CPT invariant, shows only a rapid cross-over instead of a phase transition of first order.

¹This in turn would imply the violation of Lorentz-invariance or locality or hermiticity.

²The QCD parameter $\bar{\theta}$ is the sum of the original angle of the QCD θ term and the phase of the determinant of the quark mass matrix: $\theta \rightarrow \theta + \arg \det \mathcal{M}_q$. Even if canceled by the Peccei–Quinn mechanism,^{23,24} small contributions might reappear generated by BSM physics and by possible ‘Peccei–Quinn breaking’ terms from Planck-scale physics.

Therefore, the observed matter-antimatter asymmetry together with the insufficient CP violation of the SM represents one of the few existing indicators that there might be *New Physics* beyond the Standard Model (BSM physics) which in turn might imply EDM values for subatomic particles that are larger in magnitude than those predicted by the KM-mechanism of the SM. Note that this “evidence” for substantial EDM values (especially for hadrons and nuclei) is at best circumstantial and by no means compulsory.

The current status on the experimental bound of the neutron EDM is $|d_n| < 2.9 \cdot 10^{-26} e \text{ cm}$ as measured by the Sussex/RAL/ILL group.²⁵ It cuts by two orders of magnitude into the new physics window (5), excluding in this way already some simple and minimal variants of the above mentioned New Physics models, especially some variants of (minimal) SUSY. The corresponding bound for the proton, namely $|d_p| < 2.0 \cdot 10^{-25} e \text{ cm}$, is inferred from a theoretical calculation²⁷ applied to the input from the 2016 EDM bound for the diamagnetic ^{197}Hg atom, $|d_{\text{Hg}}| < 7.4 \cdot 10^{-30} e \text{ cm}$.²⁸ The same method would predict for the neutron the bound $|d_n| < 1.6 \cdot 10^{-26} e \text{ cm}$ which is even slightly less than the Sussex/RAL/ILL limit²⁵ but is of course affected by the imponderables in extracting the relevant nuclear matrix elements of the ^{197}Hg nucleus. The EDM bound on the electron is again inferred indirectly, since theoretical calculations^{29–31} are needed to deduce it from the corresponding EDM bounds on paramagnetic atoms, e.g. ^{205}Tl with $|d_{\text{TL}}| < 9.4 \cdot 10^{-25} e \text{ cm}$,³² or on polar molecules, as e.g. YbF ^{33,34} or the recent ThO measurement by the ACME group.³⁵ The latter gives the most stringent bound on the electron EDM, $|d_e| < 8.7 \cdot 10^{-29} e \text{ cm}$.

All the measurements mentioned above have in common that they only apply to overall charge-neutral states, since the corresponding particles can be confined at rest in a trap even in the presence of reversible external electric fields (and a weak holding magnetic field) which are needed to extract the EDM signal. In order to trap charged particles (e.g. the proton, deuteron or helion), which would just be accelerated by a constant electric field, storage rings (see e.g. Refs. 36–38) or – as in the case molecular ions – traps with rotating electric fields³⁹ have to be applied. In fact, as a byproduct of $(g - 2)_\mu$ measurements in storage rings, there already exist a very weak bound on the EDM of muon,⁴⁰ $|d_\mu| < 1.8 \cdot 10^{-19} e \text{ cm}$, as compared to the SM estimate of $\sim 10^{-35} e \text{ cm}$.

3. EDM Sources Beyond the KM Mechanism

If a nonzero permanent EDM could eventually be inferred from some measurement, we would then know that the source behind the pertinent T violation (or CP violation if CPT holds) would most likely not be the KM mechanism of the SM (recall Eq. (5)), but we would still be unable to pin down the very CP violating mechanism: it could be genuine New Physics (as SUSY, two-Higgs models, left-right symmetric models) or just the QCD θ term, if the relevant angle were in the window $10^{-10} \gtrsim |\bar{\theta}| \gtrsim 10^{-14}$ (see below). In the case of genuine New Physics, the scale of the relevant CP violating operator(s) would have to be larger than the electroweak scale, probably even larger than what is accessible by LHC physics.

However, by matching possible candidate-models of CP violating physics at these high scale(s) to the coefficients of SM operators of dimension-six^k and higher, the machinery of effective field theories (EFTs) and the renormalization group can be applied.⁴² In a repeating chain, the relevant operators, which also mix under this procedure, can perturbatively be run down until a (SM) particle threshold is reached (subsequently the top quark, Higgs boson, W^\pm and Z bosons, and finally the bottom and charm quarks), where the corresponding particle should be integrated out and the coefficients of the operators should be matched to those containing only the remaining active SM degrees of freedom. This cascading perturbative procedure has to stop when the realm of non-perturbative QCD is reached somewhere between 2 GeV and 1 GeV, say. At this chiral scale Λ_χ the pertinent EFT Lagrangian of dimension-six⁴³ can be written as¹

$$\begin{aligned} \mathcal{L}^{\mathcal{F}, \mathcal{P}} = & -\bar{\theta} \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a \\ & - \frac{i}{2} \sum_{q=u,d} d_q \bar{q} \gamma_5 \sigma_{\mu\nu} F^{\mu\nu} q - \frac{i}{2} \sum_{q=u,d} \tilde{d}_q \bar{q} \gamma_5 \frac{1}{2} \lambda^a \sigma^{\mu\nu} G_{\mu\nu}^a q \\ & + \frac{d_W}{6} f_{abc} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a G_{\mu\nu}^b G_\nu^{c\rho} + \sum_{i,j,k,l=u,d} C_{ijkl}^{4q} \bar{q}_i \Gamma q_j \bar{q}_k \Gamma' q_l \quad (10) \end{aligned}$$

where we included, for completeness, the dimension-four QCD θ term as well. The relevant quantities are the quark fields of flavor q , the field strength tensors $F_{\mu\nu}$ and $G_{\mu\nu}^a$ of the photon and the gluon, respectively, the

^kThe solely existing CP violating operator of dimension five is a Majorana mass term which is only relevant for neutrino physics.⁴¹

¹In the following, we will concentrate on the EDM contributions to nucleons and light nuclei. Therefore, the terms involving leptons and strange quark contributions are not listed.

color $SU(3)$ structure constants f_{abc} and Gell-Mann matrices λ^a . The various Lorentz structures of the matrices Γ and Γ' ensure that the dimension-six four-quark operators, which have net zero flavor, violate the P and T symmetries.⁴³ The coefficients d_q of the quark EDM terms and \tilde{d}_q of the quark chromo-EDM terms scale as $\sim v_{EW}/\Lambda_T^2$ while the coefficient d_W of the gluon chromo-EDM term, the so-called Weinberg term, and the coefficients C_{ijkl}^{4q} of the four-quark EDM terms scale as $\sim 1/\Lambda_T^2$. Λ_T is the scale of the underlying T (or CP) breaking model and v_{EW} is the electroweak vacuum expectation value which is a relic of the original coupling to the Higgs field which had to be inserted to preserve the SM symmetries.^m Note that via a chiral $U_A(1)$ transformation the first term on the right hand side of Eq. (10) can be rotated into the term $-\bar{\theta}m_q^*\sum_q \bar{q}i\gamma_5 q$ where $m_q^* = m_u m_d/(m_u + m_d)$ is the reduced quark mass. In this way, it is manifest that all EDM contributions of the QCD θ term have to vanish in the chiral limit (as it is also the case for the quark and quark chromo EDM contributions) and that therefore the nucleon EDM induced by the *strong* CP breaking term has to scale as

$$|d_N^{\bar{\theta}}| \sim \bar{\theta} \cdot \frac{m_q^*}{\Lambda_{QCD}} \cdot \frac{e}{2m_N} \sim \bar{\theta} \cdot 10^{-16} e \text{ cm}, \quad (11)$$

such that the window (5) for physics beyond the KM mechanism of the SM together with the current bound on the neutron EDM²⁵ is compatible with the $10^{-10} \gtrsim |\bar{\theta}| \gtrsim 10^{-14}$ window for searches of strong CP breaking.

The θ -term contribution, the Weinberg term and two of three four-quark terms are flavor/isospin symmetric, while the quark and quark chromo EDMs can be separated into isospin-conserving and isospin-breaking combinations, respectively. The third four-quark operator stems from left-right symmetric physics which breaks isospin and also chiral symmetry at the fundamental level – for more details, see e.g. Ref. 43.

4. EDMs in the Non-Perturbative Realm of QCD

At and below the chiral scale Λ_χ , perturbative methods are not applicable any longer in order to continue towards the hadronic, nuclear or even atomic scales relevant for the EDM experiments, since the degrees of freedom change from the quark/gluon ones to hadronic ones. The relevant

^mThus the coefficients $d_{u,d}$ and $\tilde{d}_{u,d}$ effectively scale as $\sim m_{u,d}/\Lambda_T^2$ to ensure that chiral symmetry is preserved in the limit of vanishing current quark masses $m_{u,d}$. The quark and quark chromo EDM operators are therefore counted as dimension-six operators.

methods which allow for an estimate of the pertinent uncertainties are lattice QCD and chiral effective field theory (i.e. chiral perturbation theory and its extension to multi-baryon systems).

Some progress has been made in lattice QCD calculations of the EDM of the neutron (and in some cases of proton) when it is induced by the θ term, see e.g. Refs. 44,45, but the extrapolation to physical pion masses still seems to be problematic for specific lattice methods.⁴⁵ First lattice results for the quark EDM scenario relating the nucleon EDM to the tensor charges of the quark flavors are promising⁴⁶ but of course not sufficient to constrain realistic models of CP breaking. Recently, there have been attempts to work out the quark flavor contributions to the nucleon EDMs also for the case of quark chromo EDMs.⁴⁷ Lattice estimates of the nucleon EDM resulting from the Weinberg term or even from the four-quark terms are still left for future studies, not to mention lattice computations of the EDMs of (light) nuclei.

What is the situation from the chiral EFT point of view? The first chiral calculations of the nucleon EDM induced by the QCD θ term were already performed in the late seventies of the last century.⁴⁸ The results of this calculation and more modern ones^{49,50} are that the leading and sub-leading CP violating pion-loop contributions to the isovector nucleon EDM could be more and more pinned down while the isoscalar contribution turned out to be more suppressed. In fact, the leading chiral loop diverges, inducing a logarithmic scale dependence and the need for finite counter terms of the same order as the isovector loop contribution to the neutron and proton EDMs. The number of these terms can be constrained to two as shown in Ref. 50 and confirmed by Ref. 51 – even for the three-flavor case. As there do not exist any measurement or theoretical methods (apart from lattice studies for the θ -term scenario) to constrain these counter-term coefficients other than naive dimensional analysis arguments (which only refer to the magnitude but not to the sign), there is not much predictive power of the chiral EFT approach for the total single-nucleon EDMs. Only when these calculations are coupled with the lattice ones, which have their own problems as mentioned above, there can be predictions for the $\bar{\theta}$ -induced nucleon EDMs, see Refs. 51,52 and Ref. 44.

5. EDMs of Light Nuclei

However, the situation is quite different for light nuclei as e.g. the deuteron or helion. As already observed in the mid-eighties of the last century in

Ref. 53, the same CP -violating pion exchange that causes the divergence in the loop diagrams appears already at the tree-level order in the two-nucleon contributions to the EDMs of (light) nuclei, such that in this application there is no need for sizable counter-terms which are always local contact-terms by nature. In fact, the first CP -violating NN contact terms appear only at next-to-next-to-leading order relative to the contribution of the corresponding pion exchange diagram, see e.g. Ref. 54.

5.1. The hadronic parameters

Using chiral symmetry and isospin structure arguments the following chiral EFT Lagrangian for the (leading) P - and T -violating terms including single-nucleon, purely pionic, pion-nucleon and two-nucleon-contact interactions can be postulated:^{43,54–59}

$$\begin{aligned} \mathcal{L}_{\text{EFT}}^{\mathcal{TP}} = & -d_n \bar{N}(1 - \tau_3) S^\mu N v^\nu F_{\mu\nu} - d_p \bar{N}(1 + \tau_3) S^\mu N v^\nu F_{\mu\nu} \\ & + (m_N \Delta) \pi_3 \pi^2 + g_0 \bar{N} \vec{\tau} \cdot \vec{\pi} N + g_1 \bar{N} \pi_3 N \\ & + C_1 \bar{N} N \mathcal{D}_\mu (\bar{N} S^\mu N) + C_2 \bar{N} \vec{\tau} N \cdot \mathcal{D}_\mu (\bar{N} S^\mu \vec{\tau} N) \\ & + C_3 \bar{N} \tau_3 N \mathcal{D}_\mu (\bar{N} S^\mu N) + C_4 \bar{N} N \mathcal{D}_\mu (\bar{N} \tau_3 S^\mu N) \\ & + \text{terms of higher order in the chiral expansion,} \end{aligned} \quad (12)$$

where in principle the values of the coefficients of the effective Lagrangian (12) characteristically depend on the coefficients of the Lagrangian (10) (and might eventually be derived by lattice methods). In this way the models for the underlying physics, which again feed with different strength into the coefficients of (10), can in principle be disentangled if sufficiently enough EDM measurements can be matched to sufficiently enough EDM calculations of single- and multi-baryon systems. Currently, this step from the EFT Lagrangian (10) to the chiral EFT Lagrangian (12) exists only in rudimentary form for the θ -term case, allowing the determination of the CP -violating three-pion coefficient Δ and, respectively, the isospin-conserving and isospin-violating πNN coefficients g_0 and g_1 as function of $\bar{\theta}$, including uncertainties:^{58,59}

$$\begin{aligned} \Delta^\theta &= \frac{\epsilon(1-\epsilon^2)}{16F_\pi m_N} \frac{M_\pi^4}{M_K^2 - M_\pi^2} \bar{\theta} + \dots &= (-0.37 \pm 0.09) \cdot 10^{-3} \bar{\theta}, \\ g_1^\theta &= 8c_1 m_N \Delta^\theta + (0.6 \pm 1.1) \cdot 10^{-3} \bar{\theta} &= (3.4 \pm 1.5) \cdot 10^{-3} \bar{\theta}, \\ g_0^\theta &= \bar{\theta} \frac{\delta m_{np}^{str}(1-\epsilon^2)}{4F_\pi \epsilon} &= (-15.5 \pm 1.9) \cdot 10^{-3} \bar{\theta}. \end{aligned} \quad (13)$$

The involved quantities are the pion decay constant, the isospin-averaged masses of the nucleon, pion, and kaon,⁴ the strong neutron-proton mass

splitting m_{np}^{str} ,^{60,61} the quark mass ratio m_u/m_d ,⁶² which feeds into $\epsilon \equiv \frac{m_u - m_d}{m_u + m_d}$, and the ChPT coefficient c_1 (related to the nucleon sigma term).⁶³ The additional contribution to g_1^θ results from an independent chiral structure⁵⁵ and its stated value was estimated in Ref. 54.

However, even in the θ -term scenario, the other coefficients, especially the total neutron and proton EDM values d_n and d_p , but also the isospin-conserving and isospin-breaking NN -contact coefficients $C_{1,2}$ and $C_{3,4}$, respectively, can only be estimated in magnitude but not in sign, either by naive dimensional arguments or by the magnitude of the subleading loop-contributions in the case of the C_i coefficients.^{58,59} While the coefficients d_n and d_p can in principle be matched to the corresponding lattice QCD calculations which still – as already mentioned – are problematic, the estimated contributions of the C_i terms have to be treated as systematic uncertainties – even for light nuclei and even in the theoretically most simple θ -term scenario.

So far there do not exist similar relations between the other parameters of the Lagrangian (10) and the effective chiral Lagrangian (12) in the case of realistic underlying models. However, for the case of a minimal left-right symmetric model, because of its inherent isospin-breaking nature, a cross-relation between the Δ parameter and g_0 and g_1 can be established:^{43,58,59}

$$\begin{aligned} g_1^{\text{LR}} &= 8c_1 m_N \Delta^{\text{LR}} = (-7.5 \pm 2.3) \Delta^{\text{LR}}, \\ g_0^{\text{LR}} &= \frac{\delta m_{np}^{\text{str}} m_N \Delta^{\text{LR}}}{M_\pi^2} = (0.12 \pm 0.02) \Delta^{\text{LR}}. \end{aligned} \quad (14)$$

5.2. The EDMs of deuteron, helion and triton

The EDMs of the deuteron, helion and triton follow from the multiplication of the coefficients of the chiral effective Lagrangian (12) (see (13) and (14) for special cases) and the nuclear matrix elements calculated in Refs. 58,59 and listed in the Tables 1 and 2, respectively, as

$$\begin{aligned} d_D &= d_p \cdot \mathcal{F}(d_p^D) + d_n \cdot \mathcal{F}(d_n^D) + g_1 \cdot \mathcal{F}(g_1^D) + \Delta \cdot \mathcal{F}(\Delta f_{g_1}^D) \\ &\quad + \{C_3 \cdot \mathcal{F}(C_3^D) + C_4 \cdot \mathcal{F}(C_4^D)\}, \end{aligned} \quad (15)$$

$$\begin{aligned} d_{3\text{He}} &= d_p \cdot \mathcal{F}(d_p[{}^3\text{He}]) + d_n \cdot \mathcal{F}(d_n[{}^3\text{He}]) + \Delta \cdot \mathcal{F}(\Delta[{}^3\text{He}]) \\ &\quad + g_0 \cdot \mathcal{F}(g_0[{}^3\text{He}]) + g_1 \cdot \mathcal{F}(g_1[{}^3\text{He}]) + \Delta \cdot \mathcal{F}(\Delta f_{g_1}[{}^3\text{He}]) \\ &\quad + C_1 \cdot \mathcal{F}(C_1[{}^3\text{He}]) + C_2 \cdot \mathcal{F}(C_2[{}^3\text{He}]) \\ &\quad + \{C_3 \cdot \mathcal{F}(C_3[{}^3\text{He}]) + C_4 \cdot \mathcal{F}(C_4[{}^3\text{He}])\}, \end{aligned} \quad (16)$$

and the analog of Eq. (16) for the triton case, i.e. ${}^3\text{He} \rightarrow {}^3\text{H}$. The terms in curly brackets (the C_3 and C_4 contributions) are of subleading order because

of the additional isospin-breaking and can be neglected in all cases, except for an underlying model which is left-right symmetric.

The first two terms proportional to d_p and d_n are the single-nucleon contributions to the total EDMs. Since d_n and d_p can independently be determined by separate experiments, these single-nucleon terms can be subtracted from the expressions in (15) and (16) in order to determine the multi-nucleon contributions of the corresponding EDMs – just by using experimental input. The quantities proportional to Δ are either contributing to genuinely irreducible three-body interactions in the helion and triton cases (which numerically, however, turn out to be small) or to finite and momentum-transfer-dependent loop corrections (see Δf_{g_1}) of the isospin-breaking CP -violating pion exchange (proportional to g_1). These corrections are exceptionally large and add up coherently to the g_1 contributions which then factually are governed by *three different* terms with different chiral structures which will be difficult to disentangle without chiral EFT methods.

There appear less terms for the deuteron case since it is only a two-nucleon system (excluding the Δ three-body term) and since the deuteron acts as an isospin filter: the isospin-conserving (CP -violating) g_0 and $C_{1,2}$ terms are excluded since they only induce a transition to the Pauli-allowed 1P_1 intermediate states which cannot be undone by the coupling of the photon. The isospin-breaking terms, however, are allowed since the transition to the 3P_1 intermediate states can be reversed by the (isovector part of the) photon coupling.

Table 1. Contributions to the deuteron EDM calculated from the N²LO (chiral) χ EFT potential,^{64,65} the Av_{18} potential⁶⁶ and the CD-Bonn potential,⁶⁷ respectively, see Refs. 58,59 (remember that $e > 0$ here). In addition, unpublished results using the recent χ EFT potential^{68–70} with terms up to N⁴LO are presented. The tabulated values still have to be multiplied by the corresponding coefficients from the chiral Lagrangian (12) which are listed as ‘units’. Each C_i has the dimension [fm³], d_n and d_p carry the dimension [e fm], while g_1 , g_0 and Δ are dimensionless.

term	N ² LO ChPT	N ⁴ LO ChPT	Av_{18}	CD-Bonn	units
$\mathcal{F}(d_n^D)$	0.939 ± 0.009	0.936 ± 0.008	0.914	0.927	d_n
$\mathcal{F}(d_p^D)$	0.939 ± 0.009	0.936 ± 0.008	0.914	0.927	d_p
$\mathcal{F}(g_1^D)$	0.183 ± 0.017	0.182 ± 0.002	0.186	0.186	g_1 e fm
$\mathcal{F}(\Delta f_{g_1}^D)$	-0.748 ± 0.138	-0.646 ± 0.023	-0.703	-0.719	Δ e fm
$\mathcal{F}(C_3^D)$	0.05 ± 0.05	0.033 ± 0.001	–	–	C_3 e fm ⁻²
$\mathcal{F}(C_4^D)$	-0.05 ± 0.05	-0.006 ± 0.007	–	–	C_4 e fm ⁻²

Table 2. Contributions to the helion and triton EDM calculated from the N²LO (chiral) χ EFT potential,^{64,65} the Av_{18} +UIX potential^{66,71} and the CD-Bonn+TM potential,^{67,72} see Refs. 58,59. Further details as in the captions of Table 1.

term	nucleus	N ² LO ChPT	Av_{18} +UIX	CD-Bonn+TM	units
$\mathcal{F}(d_n)$	³ He	0.904 ± 0.013	0.875	0.902	d_n
	³ H	-0.030 ± 0.007	-0.051	-0.038	"
$\mathcal{F}(d_p)$	³ He	-0.029 ± 0.006	-0.050	-0.037	d_p
	³ H	0.918 ± 0.013	0.902	0.876	"
$\mathcal{F}(\Delta)$	³ He	-0.017 ± 0.006	-0.015	-0.019	Δ e fm
	³ H	-0.017 ± 0.006	-0.015	-0.018	"
$\mathcal{F}(g_0)$	³ He	0.111 ± 0.013	0.073	0.087	g_0 e fm
	³ H	-0.108 ± 0.013	-0.073	-0.085	"
$\mathcal{F}(g_1)$	³ He	0.142 ± 0.019	0.142	0.146	g_1 e fm
	³ H	0.139 ± 0.019	0.142	0.144	"
$\mathcal{F}(\Delta f_{g_1})$	³ He	-0.608 ± 0.142	-0.556	-0.586	Δ e fm
	³ H	-0.598 ± 0.141	-0.564	-0.576	"
$\mathcal{F}(C_1)$	³ He	-0.042 ± 0.017	-0.0014	-0.016	C_1 e fm ⁻²
	³ H	0.041 ± 0.016	0.0014	0.016	"
$\mathcal{F}(C_2)$	³ He	0.089 ± 0.022	0.0042	0.033	C_2 e fm ⁻²
	³ H	-0.087 ± 0.022	-0.0044	-0.032	"
$\mathcal{F}(C_3)$	³ He/ ³ H	-0.04 ± 0.03	–	–	C_3 e fm ⁻²
$\mathcal{F}(C_4)$	³ He/ ³ H	0.07 ± 0.03	–	–	C_4 e fm ⁻²

In contrast to the application of phenomenological nuclear potentials,^{66,67,71,72} the calculations using chiral potentials^{64,65} allow for the specification of uncertainties in addition to central values.^{58,59} The latter are mostly compatible with the results of the phenomenological potentials (which agree, where a comparison is possible, with the calculations of other groups^{56,73,74}), except for the short-range contact terms. These are very sensitive to the model-dependent specifics of the short-range repulsion of the phenomenological potentials – for more details see Ref. 58. We therefore refrain from showing results of these phenomenological potentials for the isospin-breaking $C_{3,4}$ contributions which are of subleading nature to start with. Finally, in Table 1 also unpublished results for the recent chiral NN potential^{68–70} with terms up to order N⁴LO are reported. The values are compatible with the older N²LO calculations but with reduced uncertainties.

6. Conclusion

Let us conclude by describing a way to identify or exclude the QCD θ term or the left-right symmetric models as the primary candidate for an

underlying CP violation beyond the KM-mechanism of the SM. This can be achieved solely via measurements of the EDMs of the neutron, proton, deuteron and helion. Note that the exclusive measurements of the single nucleon EDMs will not suffice to achieve this, since any reasonable underlying model will predict d_p and d_n to be approximately of the same magnitude and most probably of opposite sign.

However, if experimental information about $d_{3\text{He}}$ and d_n can be established, then a fit-value of the $\bar{\theta}$ angle can be extracted from Eq. (16) with input from (13), treating the small contribution of the proton and of the contact terms as systematical uncertainties. With Eq. (15) applied to this result, the nuclear part of the deuteron EDM and, if the proton EDM is measured as well (or calculated by lattice methods), also the total deuteron EDM can be predicted allowing for a test of the θ -term scenario (instead or in addition to numerical lattice tests).

Alternatively, a measurement of the neutron, proton and deuteron EDM allows to extract the $\bar{\theta}$ fit-value – again solely from experiment – and for the prediction of the total helion EDM, including uncertainties. This alternative extraction has the advantage that there are not any systematical uncertainties related to the NN -contact interactions, since these are ‘filtered out’ in the deuteron case.

The characteristic signal for the QCD θ -term scenario would be

$$d_D - 0.94(d_p + d_n) \approx -(d_{3\text{He}} - 0.9d_n) \approx \frac{1}{2}(d_{3\text{H}} - 0.9d_p). \quad (17)$$

The establishment of the last part of this relation is of course rather unlikely, since a triton EDM measurement would be necessary. At the same time we would have predictions of the coefficients Δ^θ , g_0^θ and g_1^θ (with $g_1^\theta/g_0^\theta \approx -0.2$) which can be used as input for EDM calculations of heavier nuclei.

If the dimension-four QCD θ term can be excluded – this test should always be done as the first one – then the next simplest step is to test the left-right scenario which also has a telling signal. The above described measurements, either the route of $d_{3\text{He}}$ and d_n or the route of d_D , d_n and d_p allow to extract the Δ^{LR} parameter and to predict the other alternative. The characteristic signal of the left-right model would be

$$|d_n| \approx |d_p| \ll |d_D| \quad \text{and} \quad d_D \approx d_{3\text{He}} \approx d_{3\text{H}}, \quad (18)$$

which is quite distinct from the θ -term scenario. Furthermore, the ratio $-g_1^{\text{LR}}/g_0^{\text{LR}} \gg 1$ is very different from its θ -term counterpart.

If both models can be excluded, then the measured values of d_n , d_p and d_D still allow to extract an effective coefficient g_1 which includes the Δf_{g_1}

modification. Using this as an input, a further measurement of $d_{^3\text{He}}$ would then allow to isolate the value of the coefficient g_0 . The ratio g_1/g_0 of these values should be rather different from those predicted in the θ -term and in the left-right symmetric scenarios, as otherwise one of these case could not be excluded any longer. The extracted g_1 and g_0 values can be used to predict EDMs for other nuclei, namely light nuclei as the triton or heavier ones as measured in the case of diamagnetic atoms if the calculation of the nuclear matrix elements of these heavy nuclei can eventually be done with less than 50% uncertainty, say. More details can be found in Refs. 43,58,59.

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